

$$4 + (y+3)^2 = 25$$

$$\frac{-4}{-4} \quad \frac{-4}{-4}$$

$$(y+3)^2 = 21$$

$$y+3 = \pm\sqrt{21}$$

$$y = -3 \pm \sqrt{21}$$

Determine the slope of the function at the given value of x  $x=0$

G)  $(x+2)^2 + (y+3)^2 = 25$

$$2(x+2) + 2(y+3) \cdot \frac{dy}{dx} = 0$$

$$2(0+2) + 2(-+3) \frac{dy}{dx} = 0$$

$$4 + 2(-3+\sqrt{21}+3) \frac{dy}{dx} = 0$$

$$4 + 2\sqrt{21} \frac{dy}{dx} = 0$$

$$\frac{-4}{-4} \quad \frac{-4}{-4}$$

$$2\sqrt{21} \frac{dy}{dx} = -4$$

$$4 + 2(-3-\sqrt{21}+3) \frac{dy}{dx} = 0$$

$$4 - 2\sqrt{21} \frac{dy}{dx} = 0$$

$$\frac{dy}{dx} = \frac{-4}{-2\sqrt{21}} = \frac{2}{\sqrt{21}}$$

$$\frac{dy}{dx} = -\frac{4}{2\sqrt{21}}$$

Find where the slope of the curve is undefined

H)  $x^2 + 4xy + 4y^2 - 3x = 6$

Set Denominator = 0

$$2x + (4x \frac{dy}{dx} + 4y) + 8y \frac{dy}{dx} - 3 = 0$$

$$4x + 8y = 0$$

$$\frac{4x}{4} = -\frac{8y}{4}$$

$$x = -2y$$

$$x = -2$$

$$4x \frac{dy}{dx} + 8y \frac{dy}{dx} = -2x - 4y + 3$$

$$\frac{dy}{dx} (4x + 8y) = -2x - 4y + 3$$

$$\frac{dy}{dx} = \frac{-2x - 4y + 3}{4x + 8y}$$

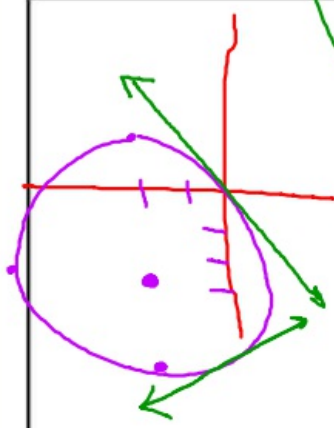
$$x^2 + 4xy + 4y^2 - 3x = 6$$

$$(-2y)^2 + 4(-2y)y + 4y^2 - 3(-2y) = 6$$

$$4y^2 - 8y^2 + 4y^2 + 6y = 6$$

$$y = 1$$

$(-2, 1)$



$y = 2 + 0(x - \sqrt{3})$   
 $y = 2$  ←  
 Perpendicular  $x = \sqrt{3}$

Find the lines that are tangent and normal to the curve at the given point

I)  $x^2 - \sqrt{3}xy + 2y^2 = 5$        $(\sqrt{3}, 2)$

$$2x - \left( \sqrt{3}x \frac{dy}{dx} + y(\sqrt{3}) \right) + 4y \frac{dy}{dx} = 0$$

$$2\sqrt{3} - \left( \sqrt{3}\sqrt{3} \frac{dy}{dx} + 2\sqrt{3} \right) + 4(2) \frac{dy}{dx} = 0$$

$$2\sqrt{3} - \frac{3dy}{dx} - 2\sqrt{3} + 8 \frac{dy}{dx} = 0$$

$$5 \frac{dy}{dx} = 0$$

$$\frac{dy}{dx} = 0$$

Find the lines that are tangent and normal to the curve at the given point

J)  $x \sin(2y) = y \cos(2x)$        $\left( \frac{\pi}{4}, \frac{\pi}{2} \right)$

$$x \cdot \left[ \cos(2y) \cdot 2 \frac{dy}{dx} \right] + \sin(2y) = y \left[ -\sin(2x) \cdot 2 \right] + \cos(2x) \frac{dy}{dx}$$

$$\frac{\pi}{4} \cdot \cos\left(\frac{2\pi}{2}\right) \cdot 2 \frac{dy}{dx} + \sin\left(\frac{2\pi}{2}\right) = -\frac{\pi}{2} \sin\left(\frac{2\pi}{4}\right) \cdot 2 + \cos\left(\frac{2\pi}{4}\right) \frac{dy}{dx}$$

$$\frac{\pi}{4} (-1) 2 \frac{dy}{dx} = -\frac{\pi}{2} (1)(2)$$

$$-\frac{2\pi}{4} \frac{dy}{dx} = -\pi$$

$$\frac{-\frac{\pi}{2} \frac{dy}{dx}}{-\frac{\pi}{2}} = \frac{-\pi}{\left(-\frac{\pi}{2}\right)}$$

$$\boxed{\frac{dy}{dx} = 2}$$

Tangent:  $y = \frac{\pi}{2} + 2(x - \frac{\pi}{4})$       Normal:  $y = \frac{\pi}{2} - \frac{1}{2}(x - \frac{\pi}{4})$

Determine the 2nd derivative of the function defined implicitly

K)  $2x^3 - 3y^2 = 8$

$$6x^2 - 6y \frac{dy}{dx} = 0$$

$$\frac{6x^2}{6y} = \frac{6y \frac{dy}{dx}}{6y}$$

$$\boxed{\frac{x^2}{y} = \frac{dy}{dx}}$$

$$\frac{d^2y}{dx^2} = \frac{y(2x) - x^2 \left(\frac{dy}{dx}\right)}{y^2}$$

$$\frac{d^2y}{dx^2} = \frac{(y)2xy - x^2 \left(\frac{x^2}{y}\right)}{y^2(y)}$$

$$\boxed{\frac{d^2y}{dx^2} = \frac{2xy^2 - x^4}{y^3}}$$

L)  $x^{\frac{1}{3}} - y^{\frac{1}{3}} = 1$

$$\frac{1}{3}x^{-2/3} - \frac{1}{3}y^{-2/3} \frac{dy}{dx} = 0$$

$$\frac{1}{3}x^{-2/3} = \frac{1}{3}y^{-2/3} \frac{dy}{dx}$$

$$\frac{\frac{1}{3}x^{-2/3}}{\frac{1}{3}y^{-2/3}} = \frac{dy}{dx}$$

$$\boxed{\frac{y^{2/3}}{x^{2/3}} = \frac{dy}{dx}}$$

$$\frac{d^2y}{dx^2} = \frac{x^{2/3} \left(\frac{2}{3}y^{-1/3}\right) \frac{dy}{dx} - y^{2/3} \left(\frac{2}{3}x^{-4/3}\right)}{\left(x^{2/3}\right)^2}$$

$$\frac{d^2y}{dx^2} = \frac{x^{2/3} \cdot \frac{2y^{-1/3}}{3} \left(\frac{y^{2/3}}{x^{2/3}}\right) - y^{2/3} \cdot \frac{2x^{-4/3}}{3}}{x^{4/3}}$$

$$\frac{(3x^{1/3}) \frac{2y^{1/3}}{3} - \frac{2y^{2/3}}{3x^{1/3}} (3x^{1/3})}{x^{4/3} (3x^{1/3})}$$

$$\boxed{\frac{2x^{1/3} y^{1/3} - 2y^{2/3}}{3x^{5/3}}}$$

$$\boxed{\frac{2\sqrt[3]{xy} - 2\sqrt[3]{y^2}}{3\sqrt[3]{x^5}}}$$